N12 • Using indices

Mathematical goals

To introduce learners to:

- fractional and negative indices.

To enable learners to:

- evaluate numerical expressions using negative and fractional indices;
- use the rules of indices with integer and fractional powers of variables.

Starting points

Learners should have some knowledge of the rules of indices for multiplying and dividing numbers in index form.

Materials required

For each learner you will need:

- mini-whiteboard.

For each small group of learners you will need:

- Card set A – *Pairs activity* (two pages).

Each page should be photocopied onto different coloured card and cut up before the session.

For each pair of learners you will need:

- Card set B – *Indices* (two pages).

Time needed

At least 45 minutes.

Suggested approach

Beginning the session

Use mini-whiteboards to revise the laws of indices by asking for possible values for the question marks in:

$2^3 \times 2^4 = 2^{?}$

$3^7 \div 3^2 = 3^{?}$

$5^{?} \times 5^{?} = 5^{12}$

$2^{?} \div 2^{?} = 2^{?}$

Working in groups (1)

Write on the board a range of calculations in powers of 2, e.g.:

$16 \div 8 = 2$

$16 \times \frac{1}{2} = 8$

$8 \times \frac{1}{4} = 2$

$\sqrt{2} \times \sqrt{2} = 2$

$8 \times 4 = 32$

$16 \div \frac{1}{4} = 64$

$8 \div 8 = 1$

$\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$

You may need to explain cube roots.

Working in pairs, learners have to rewrite the calculations in powers of 2. They must work out any that they do not know, using the rules of indices. For example, for $16 \div \frac{1}{4} = 64$:

16 is 2^4; 64 is 2^6 so $\frac{1}{4}$ must be 2^{-2} so that $2^4 \div 2^{-2} = 2^6$.

Whole group discussion (1)

Invite learners to write one of their rewritten calculations on the board and say what it tells them about indices. Check that they understand negative, fractional and zero indices by asking them to evaluate some, using mini-whiteboards.

Working in groups (2)

Ask learners to work in groups of three or four. Give each group Card set A – *Pairs activity* (i.e. 16 cards in each colour).

Learners should place all the cards face down on the table. Learners take it in turns to pick up one card of each colour and turn them over so that partners can see them. If they match, the learner keeps the pair. If they do not match, the learner places them face down on the table again. If a learner claims a pair that does not match and is correctly challenged by another member of the group, they have to put the cards back and miss a turn. The winner is the learner who has the most pairs.

Whole group discussion (2)

When the activity is finished ask each learner to explain to the whole group why they matched one of their pairs.

Working in groups (2)

Ask learners to work in pairs. Give out Card set B – *Indices* to each pair. These cards move the learning on from numerical indices to algebraic indices. Ask learners to find at least three pairs of equivalent cards but encourage them to find more and give them enough time to do so. Write on the board card-pairs that the learners have found and ask for explanations of why they are equivalent.

Next, ask learners to find sets of cards such that one card is the product of the rest. Most will find two that multiply to make the third but phrasing the task in this way allows learners who are finding it easy to be more adventurous. Again, ask for explanations.

Next, ask for sets of three cards such that the first divided by the second is equal to the third. Discuss these.

Reviewing and extending the learning

Put a number '8' in the middle of the board. Invite the whole group to suggest different numbers in index form that are equal to 8. Products and quotients can also be allowed.

Give each pair of learners a number to work on, say 9, and then share all the ideas with the whole group.

Finally, repeat the process for x^2 or some other power of x.

What learners might do next

Learners could work on differentiation and integration of functions with negative or fractional powers of x.

Further ideas

Cards can be used in this way for a range of topics that require only mental calculations e.g. fractions, surds and names of shapes.

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N12 Card set A – *Pairs activity* (page 1)

$8^{\frac{2}{3}}$	$(-1)^{\frac{1}{3}}$	$9^{\frac{3}{2}}$	$16^{\frac{1}{4}}$
$\left(\frac{1}{2}\right)^{-3}$	$\left(\frac{2}{5}\right)^{-2}$	$\left(\frac{1}{9}\right)^{-\frac{1}{2}}$	$36^{-\frac{1}{2}}$
3^{-2}	$\left(\frac{16}{25}\right)^{\frac{3}{2}}$	3^{-1}	$4^{-\frac{3}{2}}$
$8^{-\frac{1}{3}}$	$(-1)^{-2}$	$\left(\frac{1}{5}\right)^{-1}$	$\left(\frac{4}{9}\right)^{\frac{1}{2}}$

N12 Card set A – *Pairs activity* (page 2)

$\frac{2}{3}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{64}{125}$
$\frac{1}{3}$	4	1	$\frac{1}{2}$
8	−1	$\frac{1}{9}$	3
5	$\frac{25}{4}$	27	2

N12 Card set B – *Indices* (page 1)

x^{-3}	x^{3}	x^{2}	$\sqrt{x}$
$x^{\frac{3}{2}}$	$x^{\frac{1}{2}}$	$\sqrt{x^{3}}$	$x\sqrt{x}$
$x^{-\frac{1}{2}}$	$2x^{-2}$	$\dfrac{\sqrt{x}}{x}$	x^{4}
$\dfrac{1}{2}\,x^{-\frac{1}{2}}$	$\dfrac{1}{2}\,x^{-2}$	$x^{\frac{1}{3}}$	x

N12 Card set B – *Indices* (page 2)

$\sqrt[3]{x}$	x^{-1}	$2x^{-1}$	$2\sqrt{x}$
$x^{-\frac{1}{3}}$	x^{-2}	$\frac{1}{2x^2}$	$x^{\frac{2}{3}}$
x^{-4}	$\frac{1}{2\sqrt{x}}$	x^{0}	$x^{-\frac{3}{2}}$
$\frac{2}{\sqrt{x}}$	$\frac{1}{x}$	$\frac{1}{2x}$	$\sqrt[3]{x^2}$